Making meaning in mathematics problem-solving using the Reciprocal Teaching approach

Literacy Learning:
the Middle Years
Volume 22
Number 2
June 2014

Kylie Meyer | University of Southern Queensland and Queensland University of Technology, Queensland

ABSTRACT

This paper examines the application of the Reciprocal Teaching instructional approach to mathematical word problems in the middle years. The Reciprocal Teaching process is extended from the four traditional strategies of predicting, clarifying, questioning and summarising, to include further cognitive reading comprehension strategies applied to the context of solving mathematical word problems.

Introduction

Reciprocal Teaching (RT) is an evidence-based, dialogic instructional approach that has been proven to improve reading comprehension in literacy (Palincsar, 1986; Palincsar & Brown, 1983, 1984). The four comprehension strategies that traditionally constitute RT include predicting, clarifying, questioning and summarising. These four cognitive reading comprehension strategies, through the RT approach, have also been applied to support students to comprehend and solve mathematical word problems (Collen, 2011; Wessman Huber, 2011; Lamb, 2004; Quirk, 2010; Reilly, Parsons & Bortolot, 2009; Rudy, 1990; van Garderen, 2004).

Reilly et al. (2009) conducted an action research project applying RT in mathematics with two Year 7 classes in Victoria, which were streamed for ability, gender and behaviour. One class used the RT approach and the other used any problem-solving strategy of their choice. The authors had previously found that students at the same school in Victoria had experienced significant improvement in their reading ages when using the RT approach in literacy. The researchers innovated on the RT approach for problem-solving in mathematics by including predicting, clarifying, solving and summarising as their key strategies or stages.

Students were encouraged to predict the type of mathematical questions being asked, the mathematical operations required and what the answer might look like, using their prior knowledge, the structure of the text, including headings, illustrations/diagrams and problem content. During the clarifying stage, students listed words they were unfamiliar with, facts that they knew and information they had yet to determine to successfully solve the problem. In the solving phase, students used a range of problem-solving strategies and demonstrated their working using pictures, diagrams, numbers or words. Finally, during the summarising stage, students engaged in self-reflection, including justifying their answer, reflecting on how they might refine their approach if presented with a similar problem, and evaluating how they contributed to the group problem-solving task.

Students also recorded their thinking under each of the four headings: predicting, clarifying, solving and summarising. The outcomes included the observation that the RT group completed fewer problems in the allocated time period, compared to the non-RT group; however, fewer than one-third of the students in the non-RT group solved the problems correctly, whereas three-quarters of the RT group had correct solutions. The non-RT group also displayed minimal working out or checking of answers.

The RT group was encouraged to visualise the problem using concrete materials or manipulatives and these appeared to lead to better comprehension, higher student engagement and more successful outcomes. Another factor that the authors note may have led to the successful outcomes of the RT group was the fact that they had multiple engagements with the text, having to re-read the problem several times before attempting a solution.

A study of Year 5 students in New Zealand, using the RT approach to solve mathematical word problems, found that the students gained confidence when solving word problems and that both the teacher and the students found the approach useful when using a five-stage process applied to word problems focused on statistics (Quirk, 2010). The five stages in the process included: make connections, read it, plan it, solve it and check it, and aligned with Pólya's (1957) four problem-solving stages (see, plan, do, check). The difference was an added emphasis on making connections. One of the challenges that the teacher in the study identified was teaching the students to eliminate unnecessary information when identifying the most important information in the problem.

In a study of fourth grade students from two elementary schools, students were exposed over a six-week period to a modified version of RT for mathematics problem-solving. Wessman Huber (2011) found that the students using the RT approach made significantly greater changes in the pre-test to post-test results when compared to the control group. This included an increased degree of metacognition resulting from the students' abilities to describe the strategies and thinking used to solve mathematical word problems. Of note is the finding that students who received instruction in RT strategies in mathematics had 'statistically higher performance scores when compared to students in the comparison class' (p. 70).

In contrast to these positive findings, Collen's (2011) study of the application of RT to the mathematical word problem-solving skills of fifth grade students in two elementary schools in upstate New York found non-significant differences for overall post-test outcomes. To further investigate the use of Reciprocal Teaching in mathematics, this paper examines an innovation of the RT approach in mathematics. It extends the RT process to include additional cognitive reading comprehension strategies for comprehending and solving mathematical word problems.

Reading comprehension and problem-solving in mathematics

The language of mathematics can often be a formidable barrier to understanding mathematical concepts, comprehension and problem-solving (Department of Education, Employment and Workplace Relations, 2008). The challenges of comprehending the language of mathematics are further compounded for students with learning difficulties, learning support needs or those learning English as an additional language. Schell (1982) asserts that mathematics is often the most difficult content area material to read and some of the challenges of comprehending in mathematics can be attributed to its vocabulary (Chinn, 2004), as mathematics uses the standard 26 alphabet symbols, plus many non-alphabetic symbols, has differences in sentence structure to standard English prose; there is not always a one-to-one correspondence between the mathematical symbols and the spoken language required to verbalise meaning when reading mathematics, and different reading paths or directionality of text are evident (Schell, 1982). In addition to this, the order of the operations as they are written or read in mathematics is not necessarily the order in which they are carried out (Adams, 2003), and readers must comprehend the problem to determine the appropriate action to be carried out in order to solve the problem (van Garderen, 2004).

Other challenges in comprehending mathematical word problems include the need to identify sufficient, insufficient and extraneous information (Schell, 1982) and the highly technical vocabulary and multiple meanings of some everyday words in a mathematical context (Schleppegrell, 2007). Mathematical vocabulary includes technical, sub-technical, general and symbolic terms (Monroe & Panchyshyn, 1995) and knowledge of these can assist teachers to understand the cognitive demands placed on learners within mathematical contexts. Highlighting key words may not be an effective strategy in mathematical problem-solving because of the multiple meanings of words within a mathematical context (Chinn, 2004). Each strand of the mathematics curriculum has its own specialised vocabulary and, within each strand, there is also vocabulary specific to that context. It is therefore crucial that teachers apprentice students into the technical language of mathematics and the grammatical patterning that results from this technical vocabulary (Schleppegrell, 2007).

In their examination of mathematics dispositions, Gresalfi and Cobb (2006) argue for classroom instructional approaches that distribute authority and develop discipline-specific literacies that enable students to exercise agency and participate substantially and legitimately in mathematical practices. The RT approach, when applied to comprehending and solving mathematical word problems, may provide one way for this distributed authority, student agency and the development of mathematically-specific language to be afforded.

Innovating on the Reciprocal Teaching approach in mathematics

In the innovation reported in this paper, the RT approach was extended from the traditional four reading strategies used in literacy contexts (predicting, clarifying, questioning and summarising), to include other cognitive strategies, also referred as high yield strategies, specifically applied to comprehending and solving mathematical word problems. This is an innovation on a previous approach reported in this journal, which extended RT to incorporate high yield reading comprehension strategies in literacy (Meyer, 2010). A series of role cards including cues or prompts was developed in this latest innovation, to support students to engage in RT mathematics problem-solving groups. Several iterations of this approach to RT in mathematics have been developed by the author and trialled in classrooms internationally.

The approach reported is a dialogic approach to reading comprehension in mathematics, which supports learners to progress within their zones of proximal development (Vygotsky, 1978) through the use of scaffolds from the teacher, the support of peers, and prompt cards. This uses the gradual release of responsibility (Pearson & Gallagher, 1983) approach to support student independence. The premise is that teachers will:

- explicitly model each of the stages or reading strategies within a mathematical word problem-solving context;
- guide the students to apply each strategy through small group work;
- gradually release responsibility as groups work towards independence through each of the stages in the process.

This approach encourages the development of accountable talk and close reading within a mathematical problem-solving context.

The author's original iteration of the application of RT to mathematics word problem-solving and comprehension included the following stages or strategies:

- predicting;
- clarifying;
- questioning;
- visualising;
- connecting;
- calculating;
- summarising.

These stages were extended in subsequent iterations to include:

Literacy Learning:
the Middle Years
Volume 22
Number 2
June 2014

- the 'giving feedback' stage of the process;
- mini graphic organisers for each stage;
- interactive notebook scaffolds.

Figure 1 shows an overview of each of the stages and the prompts used to support students by scaffolding their learning within the process of comprehending and solving mathematical word problems. The role cards are usually printed in colour and laminated for durability for use during small group problem-solving sessions as indicated in Figure 2.

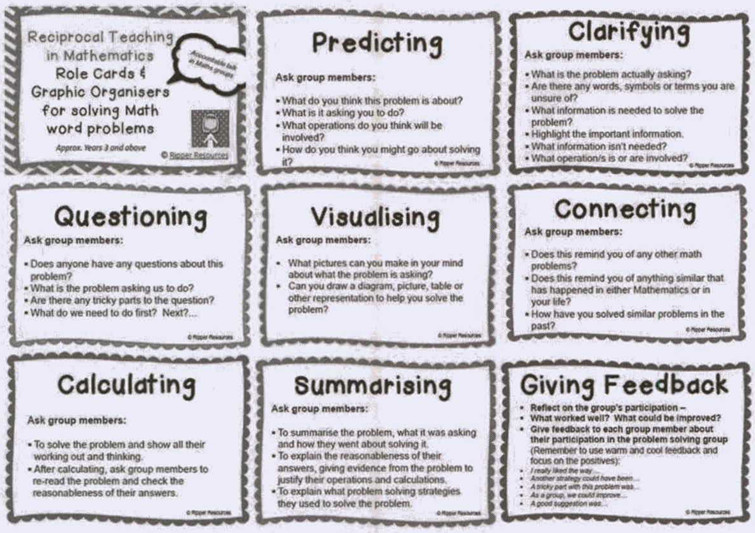

Figure 1. Reciprocal Teaching in Mathematics prompt cards

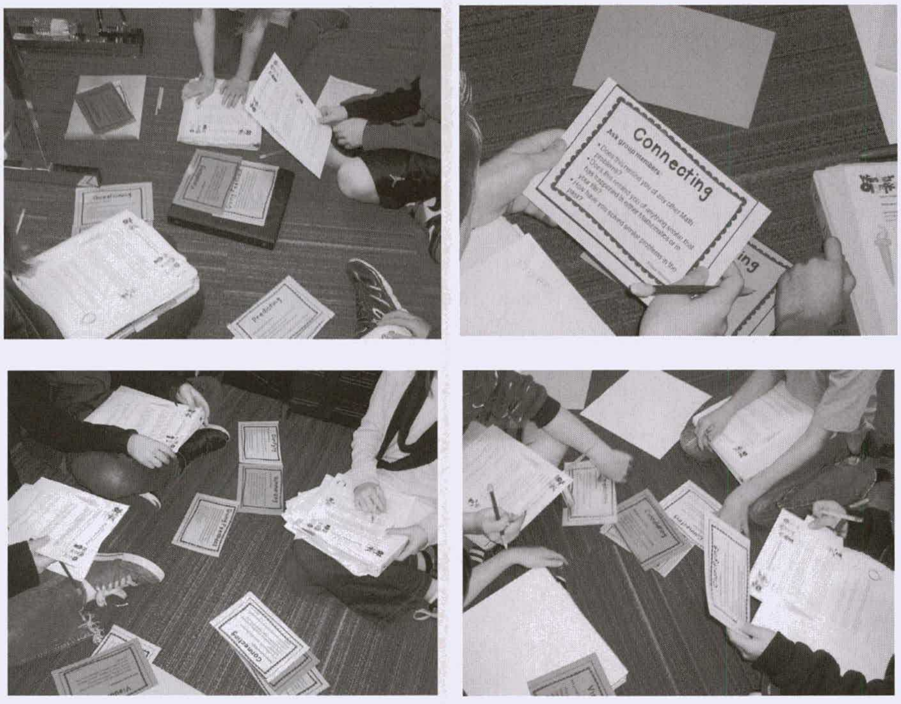

Figure 2. Reciprocal Teaching in Mathematics in action

The mini graphic organisers, as seen in Figure 3, are either photocopied multiple times for problem-solving sessions, or laminated and used with a whiteboard marker and eraser. Some teachers have used them as scaffolds on their interactive whiteboards to reduce photocopying costs. The students record their thinking in their maths journals using the interactive notebook prompts shown in Figure 4. More information about the published versions of these RT and mathematics role cards, graphic organisers and interactive notebook pages can be found at http://ripperresources.blogspot.com.au/2014/01/reciprocal-teaching.html

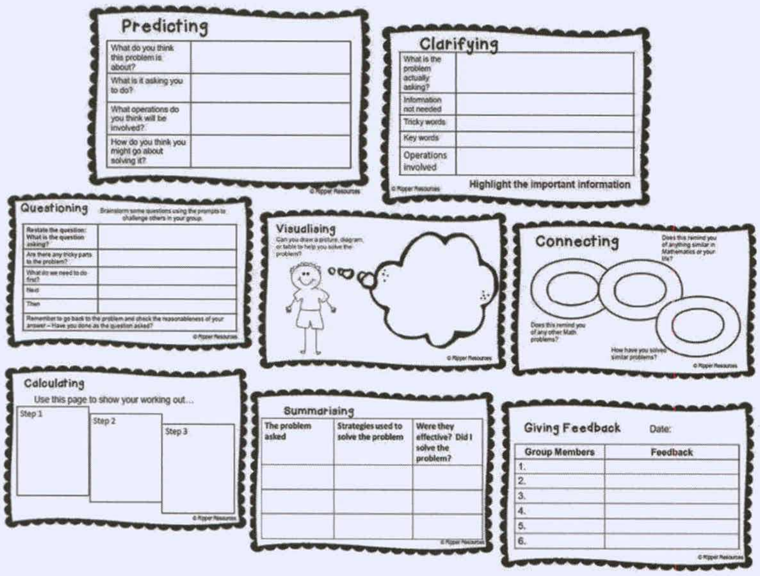

Figure 3. Reciprocal Teaching in Mathematics graphic organisers

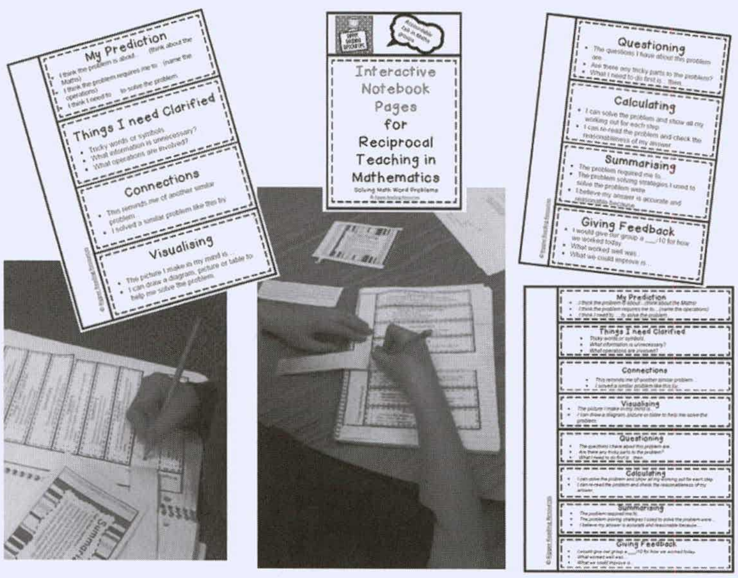

Figure 4. Reciprocal Teaching in Mathematics interactive notebook scaffolds

The innovation on the RT approach to mathematics reported here was developed prior to the review of the other studies cited earlier in this paper. Commonalities with other approaches to RT in mathematics can be identified, however, including the focus on making connections (Quirk, 2010), visualising the problem, the solving phase in the process, the summarising stage, which includes justifying answers and solutions and students evaluating their participation in group problem-solving sessions (Reilly et al., 2009), and the students' multiple engagements and re-readings of the text (Wessman Huber, 2011), which result in close reading of the mathematical word problem. The point of difference, however, is the explicit inclusion of the cognitive strategies of visualising, connecting, calculating and summarising (including explaining the reasonableness of solutions and restating the problem in the students' own words). In relation to higher order thinking, Brown and Campione (1986) state that:

> Understanding is more likely to occur when a student is required to explain, elaborate, or defend his or her position to others; the burden of explanation is often the push needed to make him or her evaluate, integrate, and elaborate knowledge in new ways. (p. 1066)

The approach to RT reported in this paper is therefore a heuristic that supports students as they learn to perform higher-level operations. It uses scaffolds for teaching higher-level cognitive strategies and provides feedback to students as an important part of the teaching and learning of these cognitive strategies (Rosenshine & Meister, 1992).

Findings: Reporting on anecdotal feedback

Several teachers have trialled the approach to RT reported in this paper, using the scaffolds provided (the role cards, graphic organisers and interactive notebook scaffolds). A Year 5 teacher in the United States reported that in comparison to other students in the same year level at her school:

> On every single Science and Math test this year my class has scored higher than the other two 5th grade classes; sometimes quite significantly ... I think we consistently score higher on the weekly and monthly classroom tests because my students know that there is the expectation of looking below the surface and applying problem-solving skills.

The teacher attributes this outcome to the use of Reciprocal Teaching within her classroom, including the scaffolds shared in this paper. However, further research would need to be conducted to confirm or disprove whether this is empirically the case. Other feedback has been in relation to the approach supporting students' accountability and ownership of their learning, their independent thinking – that it supported students to have an 'equal voice' in group work – and further that the approach 'empowers students' and helps develop their vocabulary and reading comprehension strategies.

Conclusion

This paper has reported on one innovation that applied the Reciprocal Teaching approach to small group mathematical problem-solving groups, in order to support students to comprehend and solve mathematics word problems. Previous studies that have incorporated innovations on RT in mathematics were discussed and the author's own innovation on RT in mathematics was described. Further research is required to ascertain empirically if the approach reported in this paper does have an effect on improving reading comprehension of mathematical word problems and/or problem-solving in mathematics. However, at this early stage, the anecdotal feedback is promising and the findings from previous published studies are encouraging.

Acknowledgements

Many thanks to Joy Hollander, a fifth grade teacher from Las Vegas, Nevada, USA, and another fifth grade teacher, who wishes to remain anonymous, from Utah, USA. They provided photographs taken in their classrooms while they were using the RT resources reported in this paper.

Clip art and fonts for the RT resources developed by the author and reported in this paper were provided by:

- Colleen Alaniz (http://www.teacherspayteachers.com/Product/Clip-Art-Cuties-757701);
- Graphics by Sarah Beth (http://www.teacherspayteachers.com/Product/Callouts-Collection-Clipart-Graphics-Speech-Thought-Bubbles-727451);
- Graphics from the Pond (http://www.teacherspayteachers.com/Product/Fuzzy-Fun-Scalloped-Borders-FREE-Graphics-From-the-Pond-641474 (http://www.teacherspayteachers.com/Product/Flip-Flap-Template-Set-Decorative-Great-for-Interactive-Notebooks-997510);
- Jen Jones Hello Fonts (http://www.teacherspayteachers.com/Product/Hello-Fonts-License-for-Commercial-Use-Single-License-Single-Designer-395172);

- Julie O'Connor (http://www.teacherspayteachers.com/Product/Speech-Bubbles-clip-art-and-instructional-tool-805414);
- nahhan73 (http://www.etsy.com/au/shop/nahhan73?section_id=10804022)
- Zip-a-Dee-Doo-Dah Designs (http://www.teacherspayteachers.com/Product/Rainbow-Links-Chains-Clip-art-Over-30-Graphics-690697).

References

Adams, T.L. (2003). Reading mathematics: More than words can say. *The Reading Teacher, 56*(8), 786–795.

Brown, A.L. & Campione, J.C. (1986). Psychological theory and the study of learning disabilities. *American Psychologist, 41*(10), 1059–1068.

Chinn, S. (2004). The language of maths. In S. Chinn (Ed.), *The trouble with maths: A practical guide to helping learners with numeracy difficulties* (pp. 93–104). London: Routledge Falmer.

Collen, M.H. (2011). *Fifth grade children's use of reciprocal teaching to solve word problems in mathematics* (Doctoral thesis, State University of New York at Albany). Retrieved from ProQuest Dissertations & Theses A&I database.

Department of Education, Employment and Workplace Relations. (2008, May). *National numeracy review report.* Retrieved 14.2.14, from http://www.coag.gov.au/sites/default/files/national_numeracy_review.pdf

Gresalfi, M.S. & Cobb, P. (2006). Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. *Pedagogies: An International Journal, 1*(1), 49–57. doi:10.1207/s15544818ped0101_8

Lamb, J. (2004). *Reciprocal Teaching in the mathematics classroom: Improving student achievement through reading* (Master of Arts in Education thesis, Pacific Lutheran University, Tacoma, WA). Retrieved from ProQuest Dissertations & Theses A&I database.

Meyer, K. (2010). 'Diving into reading': Revisiting reciprocal teaching in the middle years. *Literacy Learning: the Middle Years, 18*(1), 41–52.

Monroe, E.E. & Panchyshyn, R. (1995). Vocabulary considerations for teaching mathematics. *Childhood Education, 72*(2), 80–83.

Palincsar, A.S. (1986). Metacognitive strategy instruction. *Exceptional Children, 53*(2), 118–124.

Palincsar, A.S. & Brown, A.L. (1983). *Reciprocal teaching of comprehension-monitoring activities* (Technical report No. 269). Retrieved from http://gateway.library.qut.edu.au/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED225135&site=ehost-live

Palincsar, A.S. & Brown, A.L. (1984). Reciprocal teaching of comprehension-fostering and comprehension-monitoring activities. *Cognition and Instruction, 1*(2), 117–175.

Pearson, P.D. & Gallagher, M.C. (1983, October 1). *The instruction of reading comprehension* (Technical Report No. 297). Retrieved from http://gateway.library.qut.edu.au/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED236565&site=ehost-live

Pólya, G. (1957). *How to solve it: A new aspect of mathematical method* (2nd ed.). Garden City, NY: Doubleday.

Quirk, P.J. (2010). *Using reciprocal teaching and learning methods to enhance comprehension in mathematics word problems* (Master of Education thesis, Massey University, Palmerston North, New Zealand). Retrieved from http://mro.massey.ac.nz/handle/10179/2247

Reilly, Y., Parsons, J. & Bortolot, E. (2009). Reciprocal Teaching in mathematics. *Proceedings of the Mathematics of Prime Importance Conference,* the 46th Conference of the Mathematical Association of Victoria (pp. 182–189). Retrieved from http://www.mav.vic.edu.au/files/conferences/2009/13Reilly.pdf

Rosenshine, B. & Meister, C. (1992). The use of scaffolds for teaching higher-level cognitive strategies. *Educational Leadership, 49*(7), 26–33.

Rudy, D.W. (1990). *The effects of metacognitive strategies and social interaction upon mathematics learning: A reciprocal teaching study* (Doctoral thesis, Loyola University of Chicago, Chicago, IL). Retrieved from ProQuest Dissertations & Theses A&I database.

Schell, V.J. (1982). Learning partners: Reading and mathematics. *The Reading Teacher, 35*(5), 544–548.

Schleppegrell, M.J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading and Writing Quarterly, 23*(2), 139–159.

van Garderen, D. (2004). Reciprocal teaching as a comprehension strategy for understanding mathematical word problems. *Reading and Writing Quarterly, 20*(2), 225–229.

Vygotsky, L.S. (1978). *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner & E. Soubermann, Trans.). Cambridge, MA.: Harvard University Press.

Literacy Learning:
the Middle Years
Volume 22
Number 2
June 2014

Wessman Huber, C. (2011). *The impact of Reciprocal Teaching on mathematics problem solving for Grade 4 students* (Doctoral thesis, Central Connecticut State University, New Britain, CT). Retrieved from ProQuest Dissertations & Theses A&I database.

Kylie Meyer is a lecturer at the University of Southern Queensland, Springfield Campus, Queensland. She is currently completing her PhD study around conceptions of intellectual demand in the teaching of reading across the curriculum. She is passionate about literacy education in the middle years.